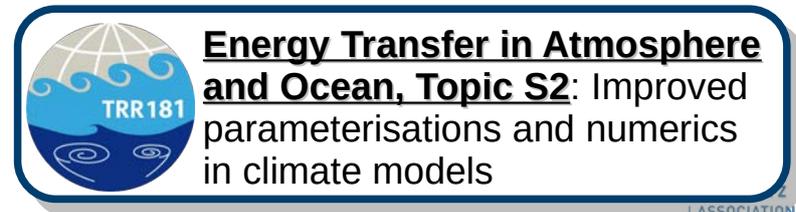


# FESOM<sup>\*</sup>2.0

## New vertical geometric features implemented into FESOM2.0

Patrick Scholz ([Patrick.Scholz@awi.de](mailto:Patrick.Scholz@awi.de)), Dmitry Sidorenko,  
Sergey Danilov, Nikolay Koldunov



# FESOM2.0 using ALE

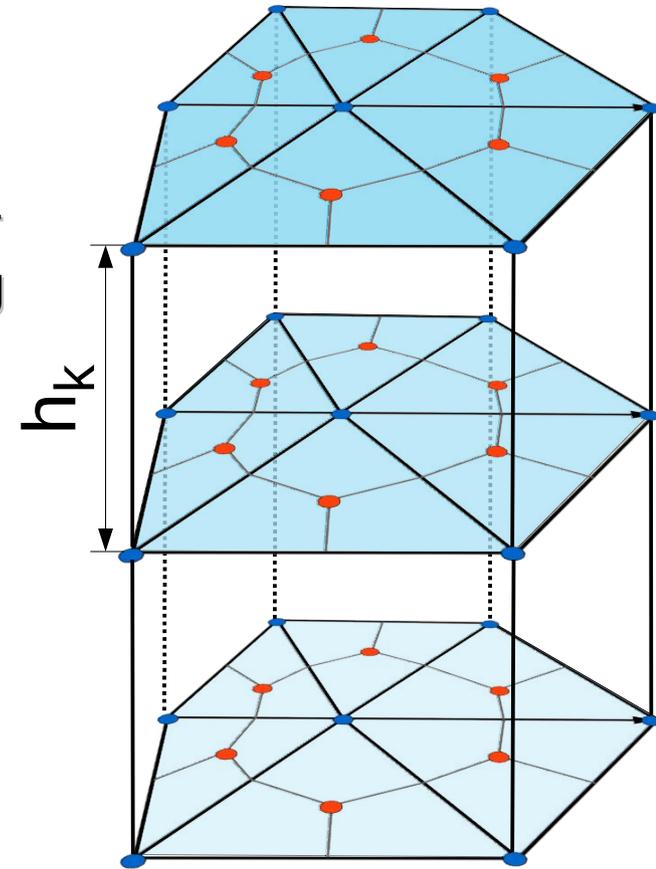
- FESOM2.0 uses **A**rbitrary **L**agrangian-**E**ulerian (**ALE**) vertical coordinate approach

## What is ALE?

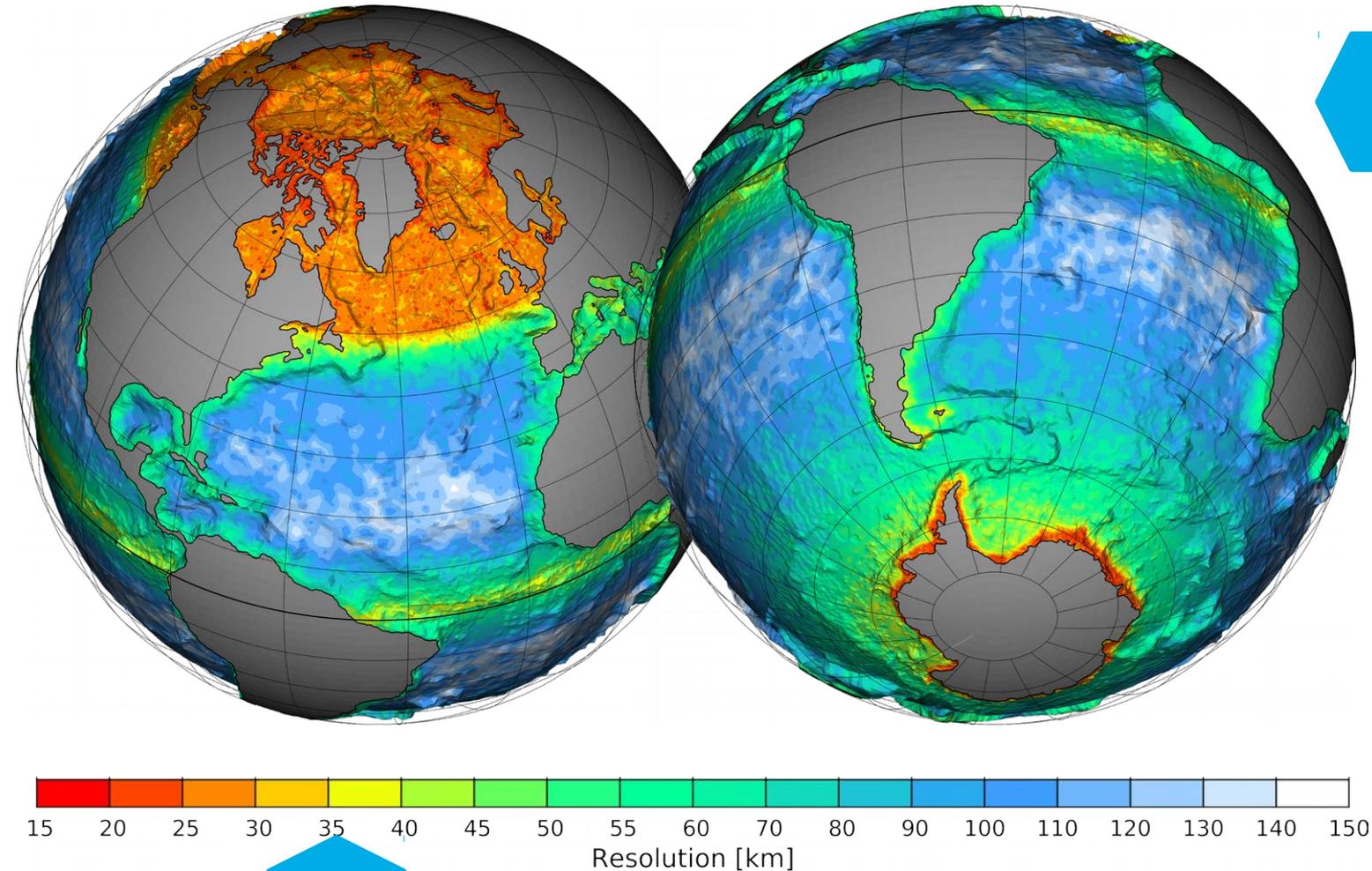
- Achieved by introducing 3d layerthickness  $h(x,y,z,t)$  as a new variable which can be fixed in time or vary under certain constrains
- It allows vertical levels to be fixed in time (Eulerian) or to move with the flow (Lagrangian) or to be something in between
- ALE has the potential to reduce unwanted spurious cross isopycnal mixing effects

## What can be implemented via ALE?

- Vertical discretisations for linear free surface (linfs), and full free surface (zlevel, zstar, ztilde)
- partial bottom cells, embedded sea ice, cavities
- Vanishing-Quasi-Sigma (VQS) coordinates, hybrid coordinates, isopycnal following coordinates.



# Model Setup



## Forcing

- Coordinated Ocean-ice Reference Experiments data set, version 2 (Large and Yeager 2009)
- Period 1948-2009
- Applied 3 spinup cycles

## Initialisation

- Polar science center Hydrographic Climatology (PHC3.0)

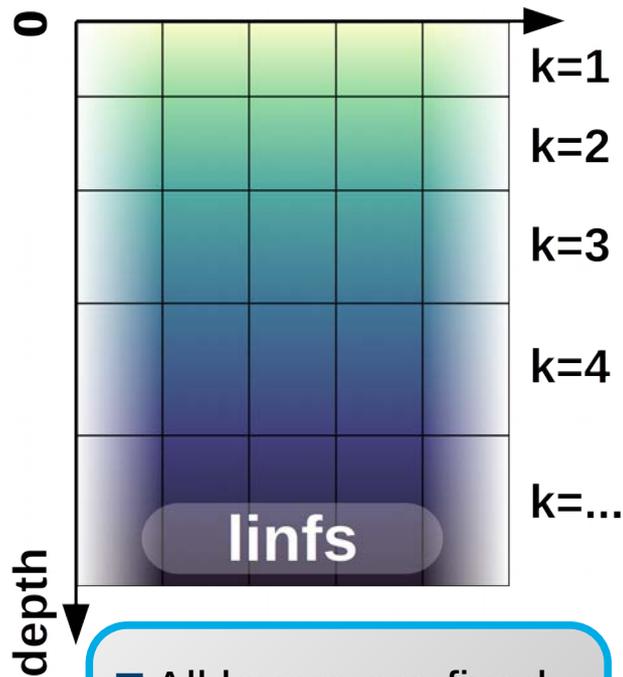
## Mesh

- CORE2 mesh, minimum resolution ~20 km (#surface vertices: 126858, #layers: 48)
- Also used for CORE2 intercomparison project with FESOM1.4



# Linear- / Full free surface

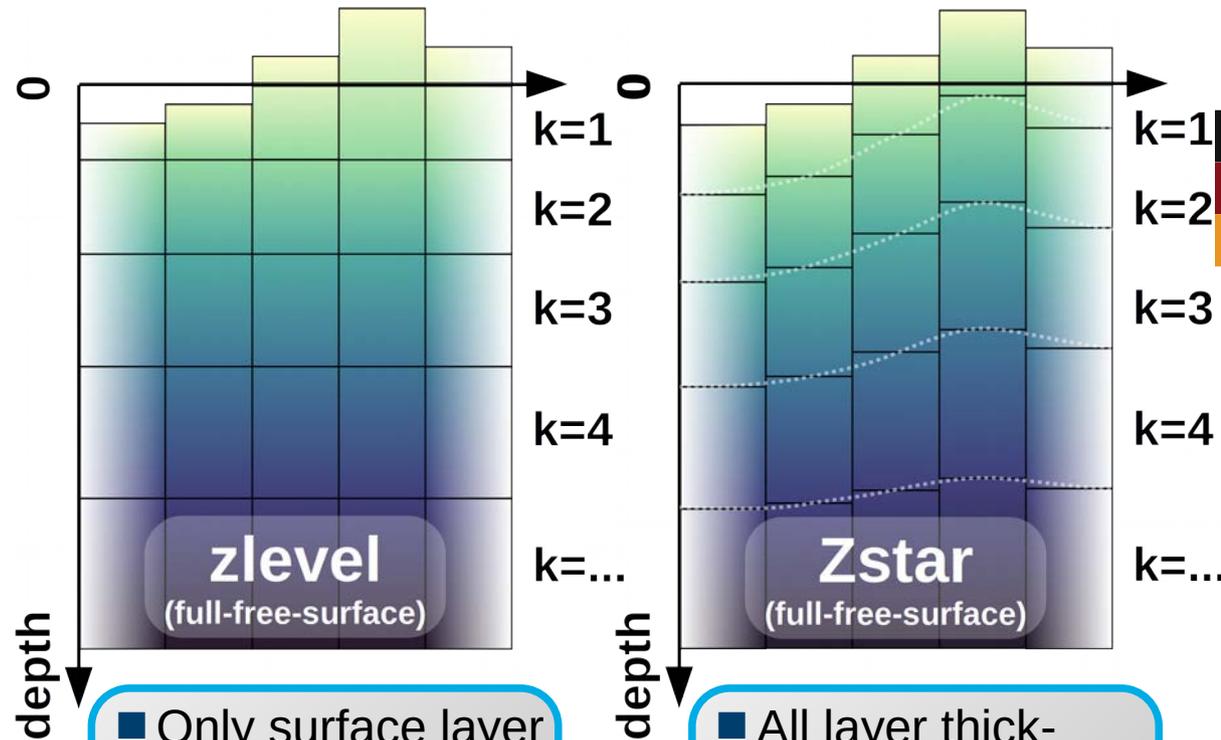
## Linear free surface



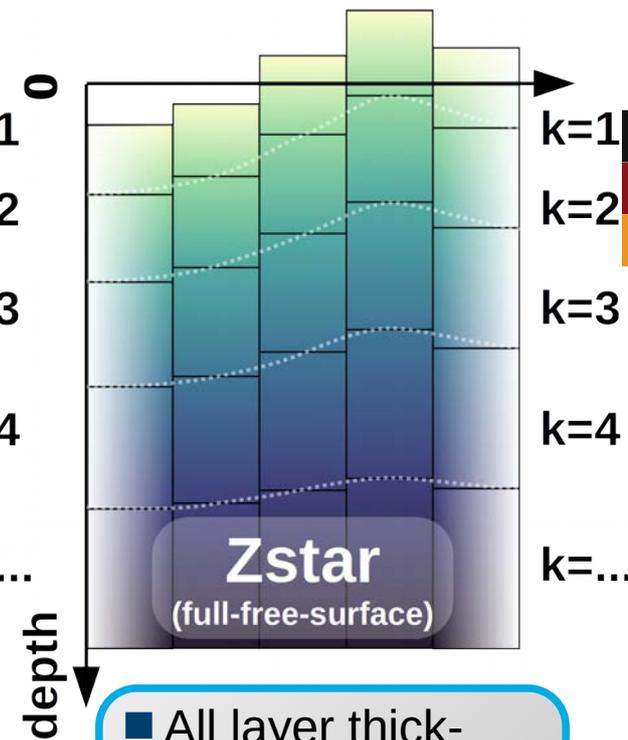
- All layers are fixed

- Freshwater flux can not be taken into account it contributes by virtual salinity flux

## Full (nonlinear) free surface



- Only surface layer thickness variates with dssh



- All layer thicknesses variate with dssh

- Freshwater flux directly affects the volume of the first layer and thus alternates salinity

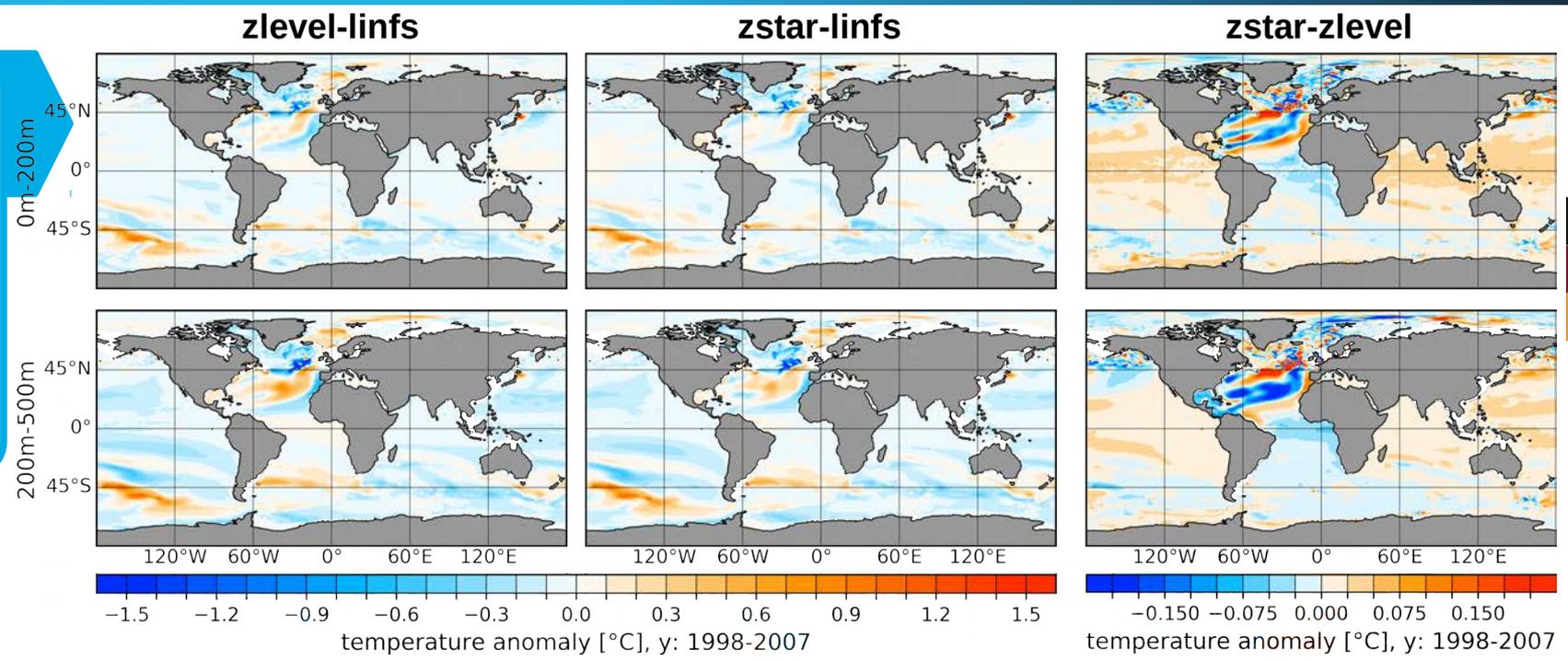
```
namelist.config: which_ALE =
  'lins', 'zlevel', 'zstar'
```



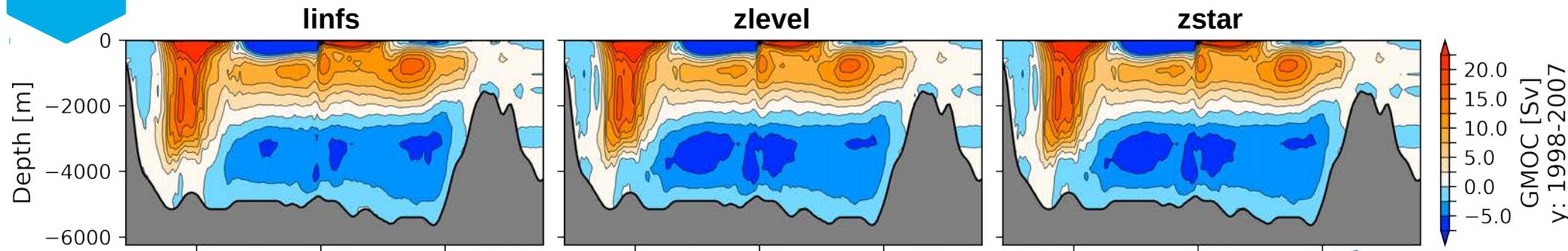


# Linear- / Full free surface

Temperature bias between zlevel/zstar and linfs (left + middle) and between zlevel and zstar (right panel)



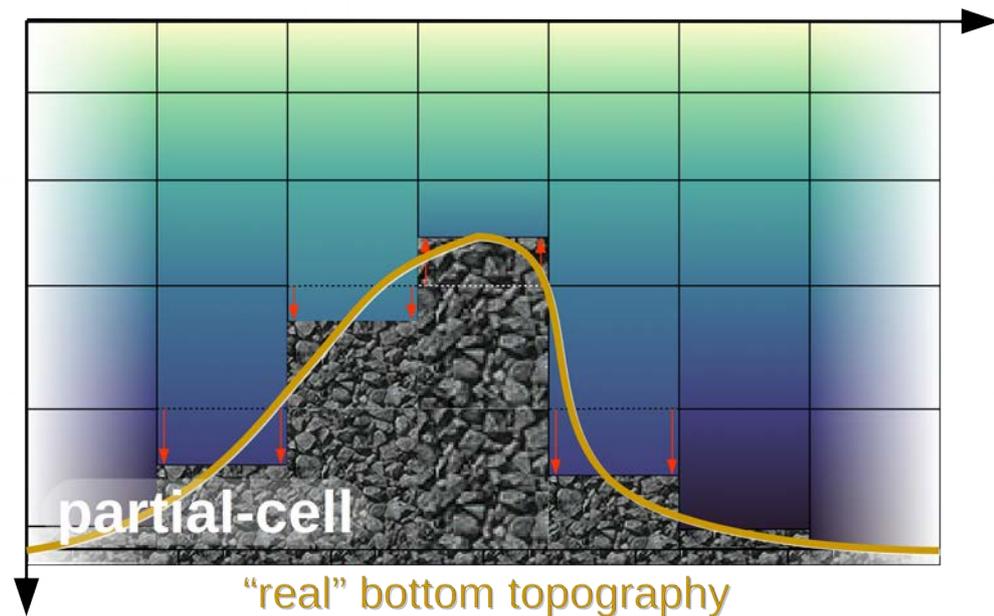
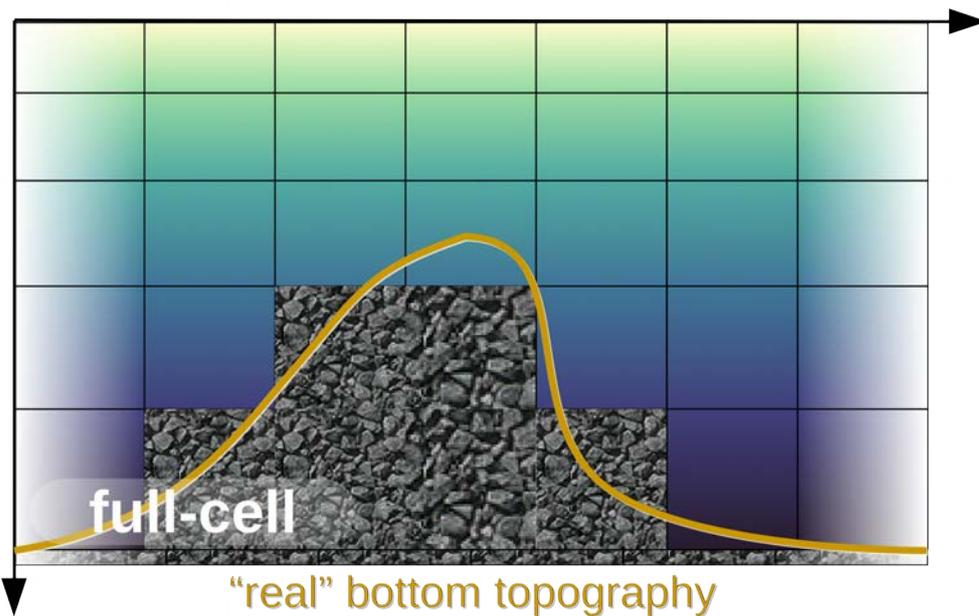
Global Meridional Overturning Circulation (MOC) for linfs, zlevel and zstar



# Partial Cells

- Adapt bottom layer thickness away from full cell thickness towards a more realistic bottom representation (namelist.config: use\_partial\_cell=.true.)
- Bottom thickness limitation:

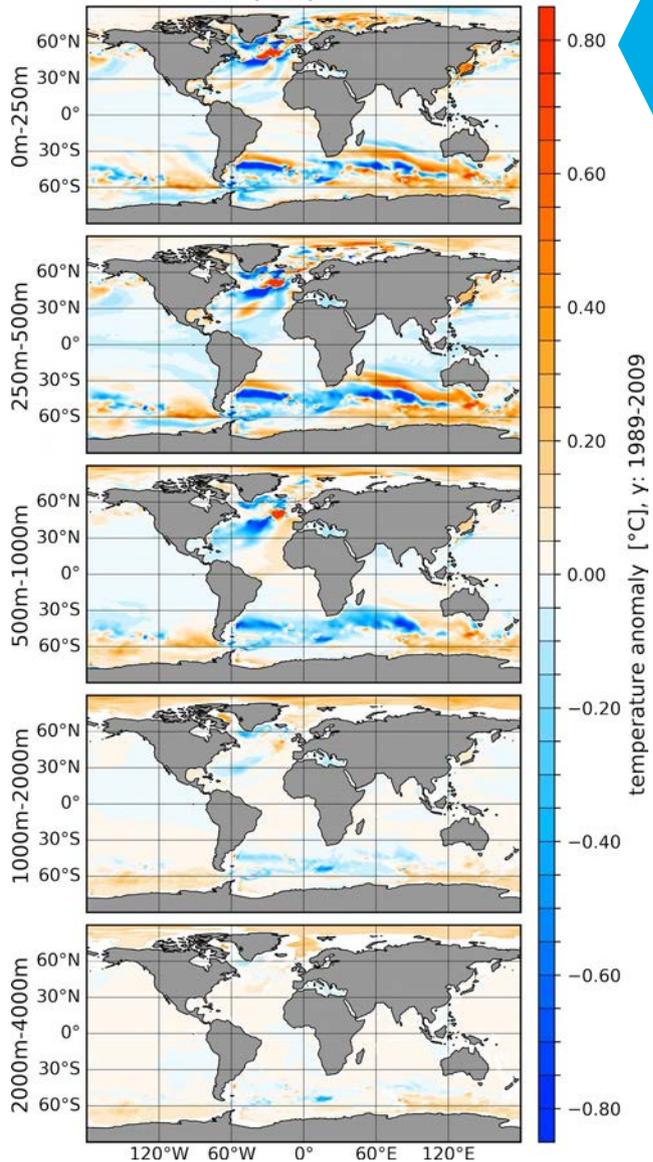
$$0.5 * h_{\text{bot, full cell}} \leq h_{\text{bot, partial cell}} \leq 1.5 * h_{\text{bot, full cell}}$$





# Partial Cells

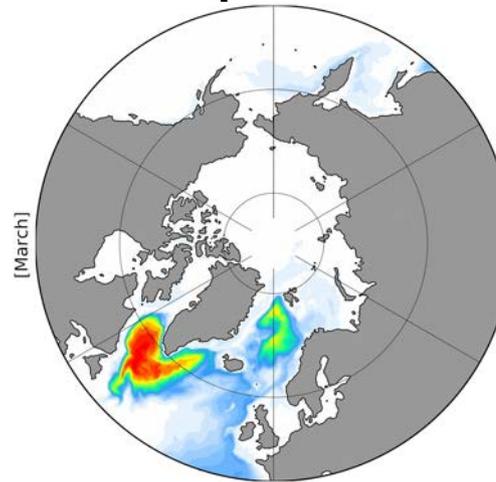
pc:1-pc:0



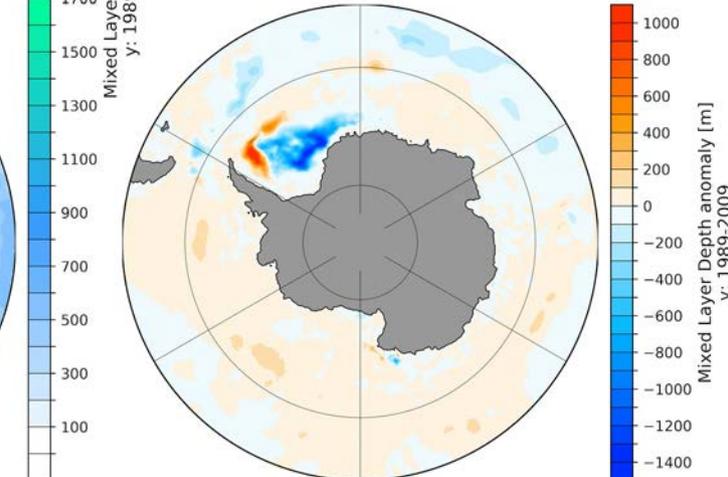
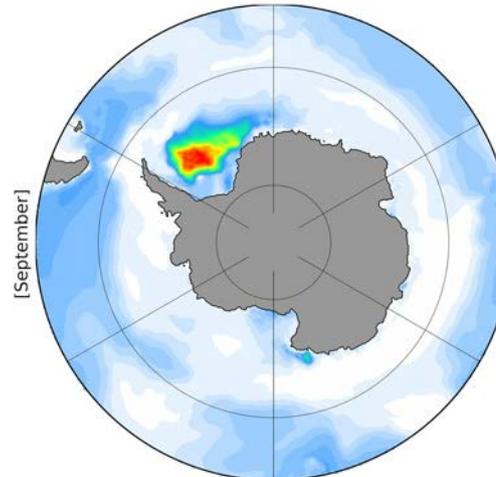
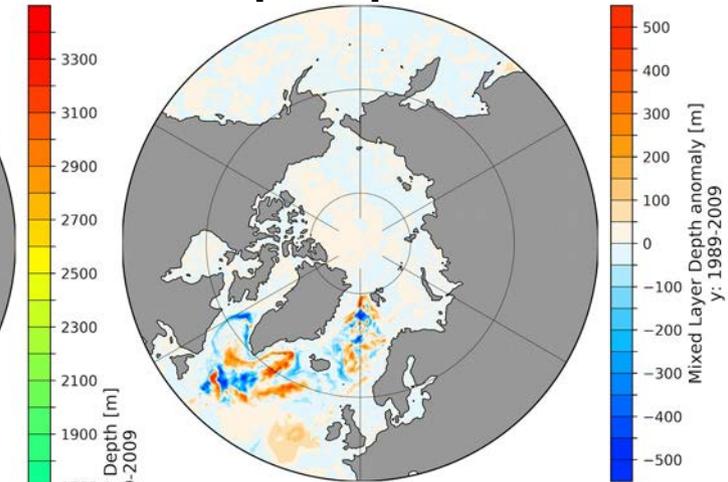
■ Mean temp. anomaly: with minus without partial cells

■ Absolute mixed layer depth (MLD) without partial cells and MLD anomaly: with minus without partial cells

pc:0

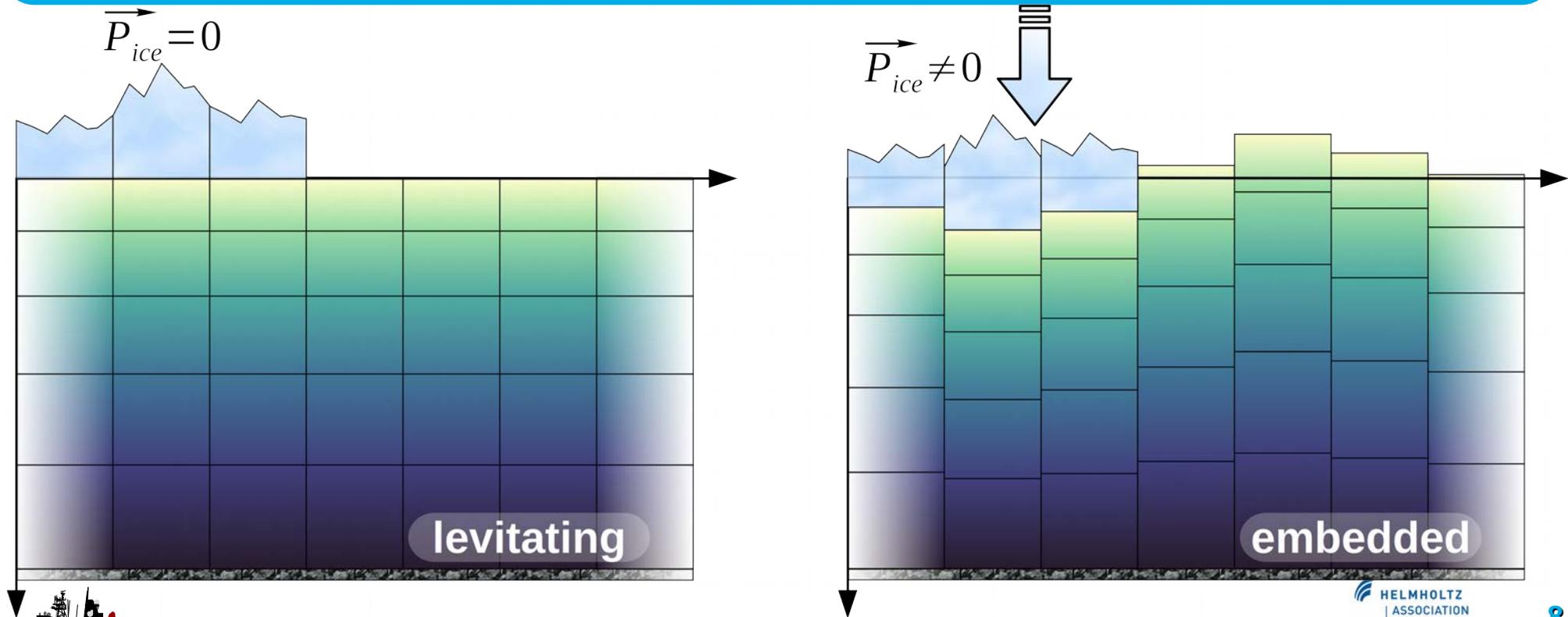


pc:1-pc:0



# Embedded sea ice

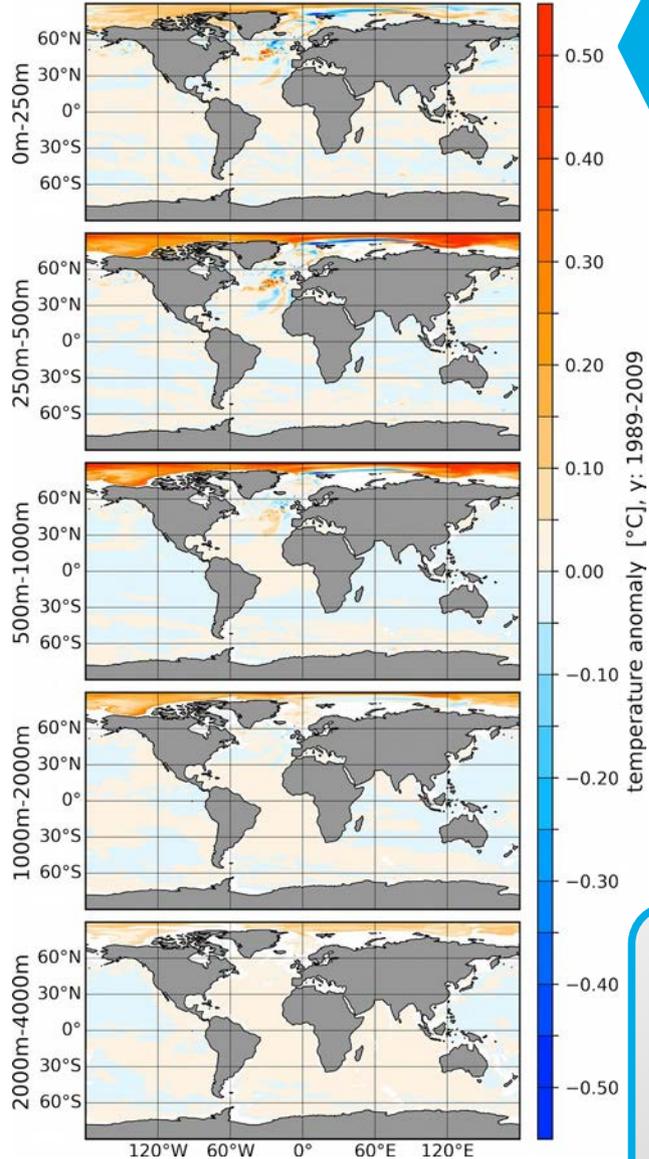
- Add pressure from sea ice loading to hydrostatic pressure (namelist.config: use\_floatice = .true.)
- freezing and melting have no effect on the oceanic pressure but divergence of the ice transport, modifying the ice-loading fields does contribute to the hydrostatic pressure
- sea ice dynamics in combination with the ice-loading coupling can be a source of oceanic variability especially near the ice edge where ice divergence/convergence is large (Campin et al., 2008)





# Embedded sea ice

## embedded-levitating



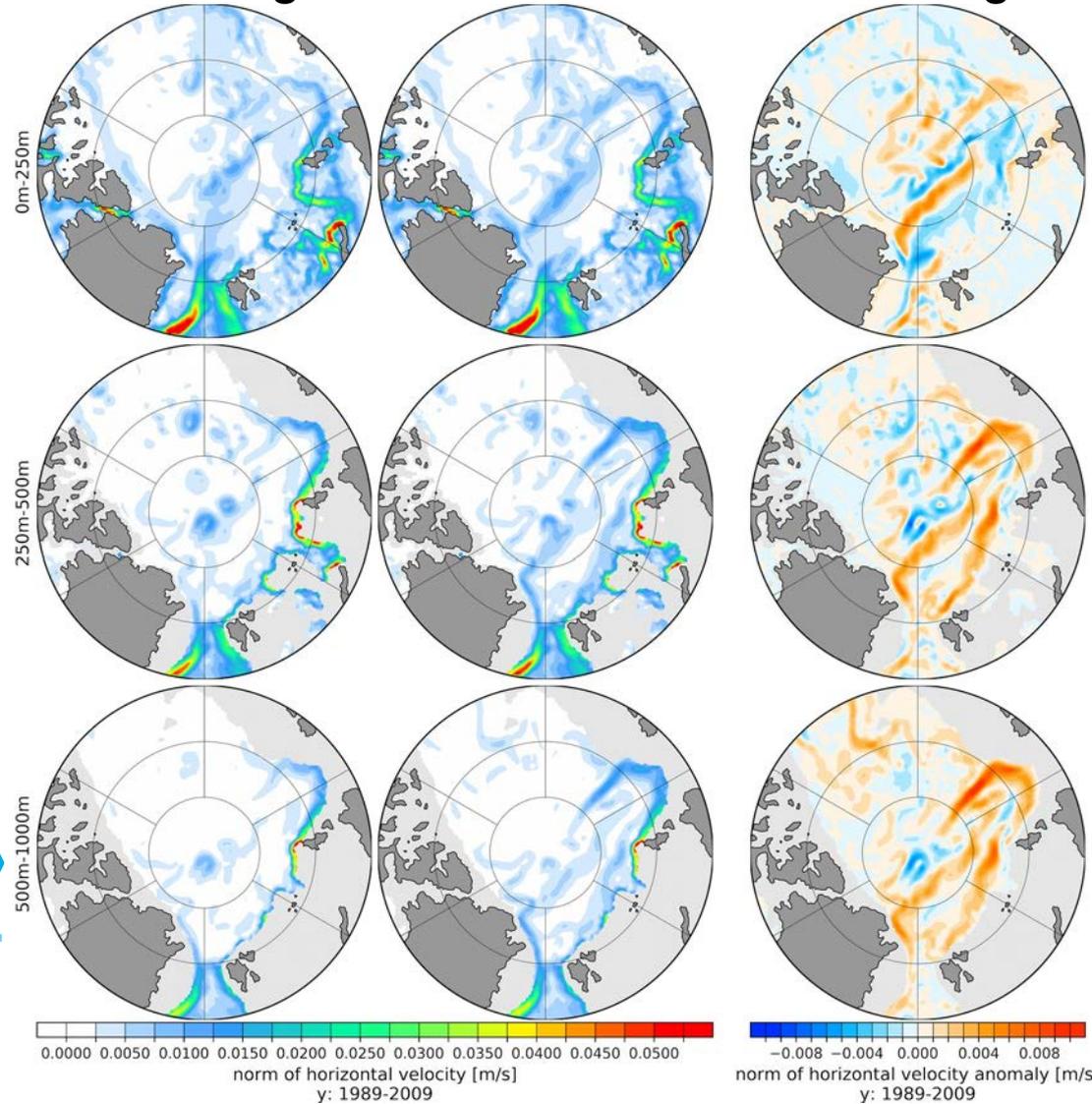
■ Mean temp. difference between floating and levitating sea ice

■ Norm of horiz. ocean velocity for levitating and embedded sea ice

## levitating

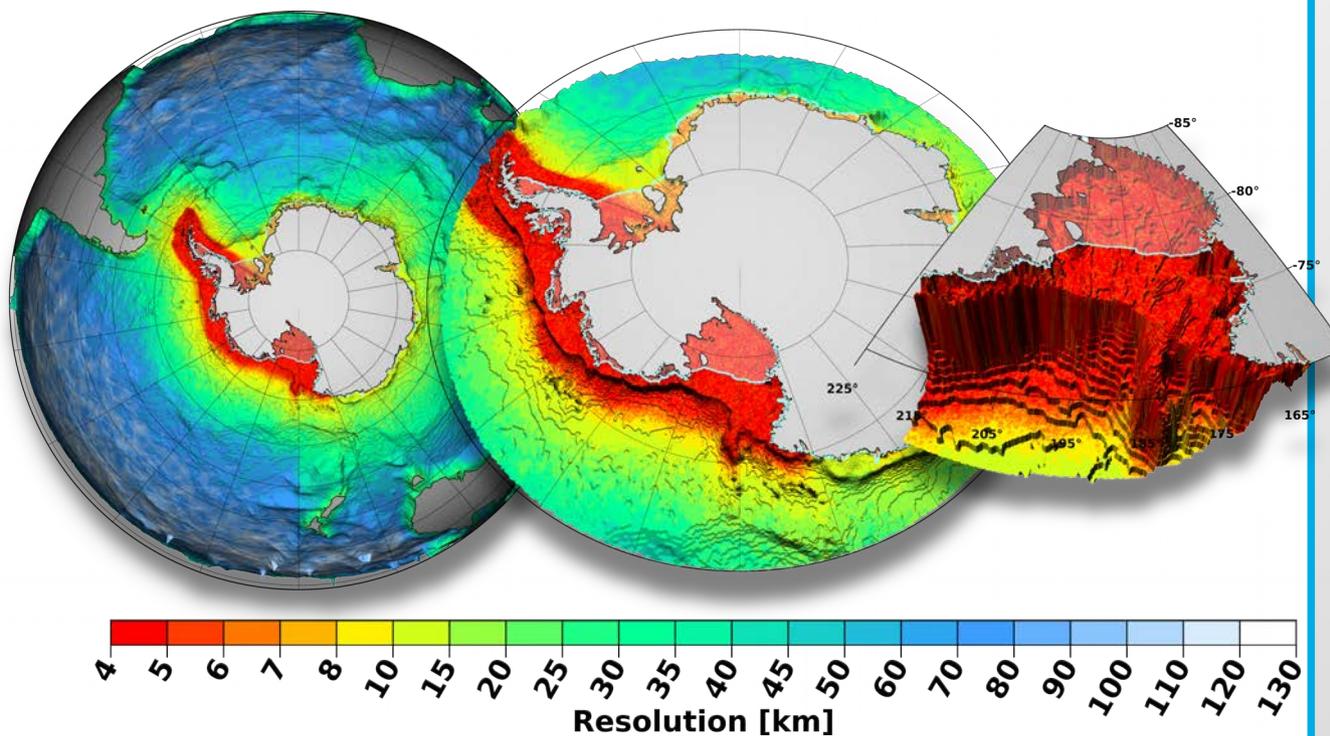
## embedded

## embedded-levitating



# Cavity

- Added ice cavity in FESOM2.0 only for geopotential vertical coordinates (terrain following & vanishing quasi sigma functionality will follow during 2<sup>nd</sup> phase of TRR181, `namelist.config: use_cavity=.true.` )



- Only need to provide one file `cavity_depth.out` with cavity geometry during partitioning
- To improve cavity-ocean boundary, surface partial cells possible at ocean-cavity edge (`use_cavity_partial_cell=.true.`)
- Heat flux, fresh-water flux and surface-stress parameterization at ocean-cavity boundary are taken from Ralph Timmermann's FESOM1.4 cavity implementation



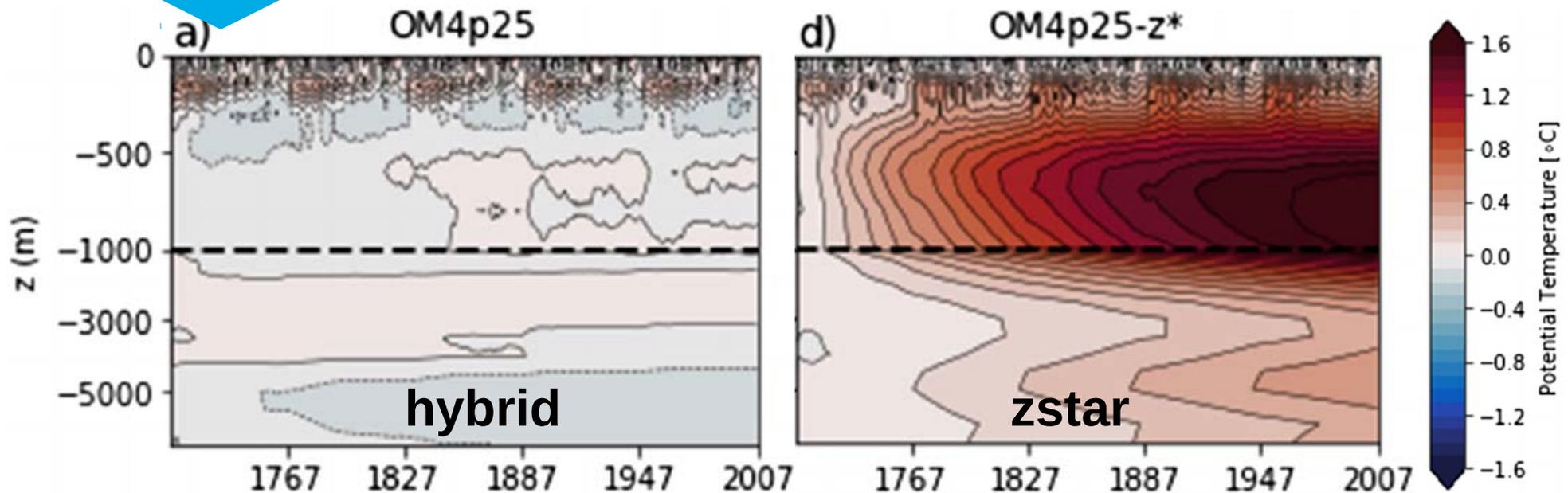
# Outlook

## ■ Geometric features to come

- Terrain following coordinates, vanishing quasi sigma (VQS) hybrid coordinates
- Long term goal: Relaxation of layer thicknesses towards isopycnal layers and combine with hybrid coordinates



- GFDL OM4 ocean sea ice model (ocean component uses MOM6,  $\frac{1}{4}$  degree resolution) using hybrid (OM4p25) and zstar (OMp25-z\*) vertical coordinates.
- Shown is temperature drift with respect to first year of simulation
- Adcroft et al. 2019, 10.1029/2019MS001726





# Finish



# FES @ M2.0

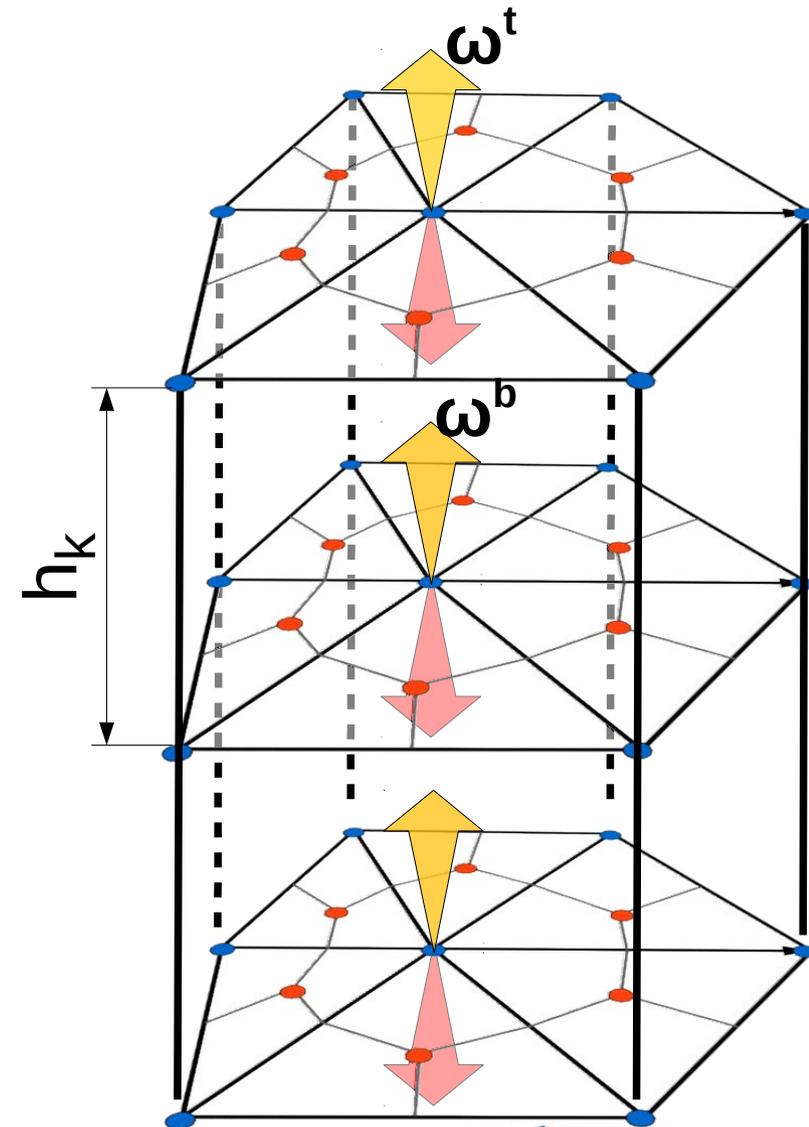


# Implementation of ALE vertical coordinates

## How we introduce ALE ?

- Introduce space-time dependent layer thickness  $h_k = h_k(x, y, t)$ , ( $k$ ...layer index),
- $h_k$  becomes part of continuity-, tracer-, momentum- and elevation equation by integrating vertically
- Introduce transport velocities  $\omega^t$  and  $\omega^b$  through top and bottom boundary
- The different vertical ALE coordinates are implemented via the continuity equation for the layer thickness

S. Danilov et al. 2017



# ALE vertical coordinates

## ■ Layer Equations:

S. Danilov et al. 2017

- Equation of motion, continuity- and tracer equation are integrated vertically over the layers → continuity equation becomes the equation on layer thicknesses

$$\partial_t h_k + \nabla \cdot (\mathbf{u}h)_k + \left( w^t - w^b \right)_k + W \delta_{k1} = 0, \quad (3)$$

- the tracer equation becomes

$$\begin{aligned} \partial_t (hT)_k + \nabla \cdot (\mathbf{u}hT)_k + \left( w^t T^t - w^b T^b \right)_k + WT_W \delta_{k1} \\ = \nabla_3 \cdot h_k \mathbf{K} \nabla_3 T_k. \end{aligned} \quad (4)$$

- Integrating eq. (3) vertically and assuming  $w^t = 0$  at the free surface → obtain the elevation equation

$$\partial_t \eta + \nabla \cdot \sum_k h_k \mathbf{u}_k + W = 0. \quad (5)$$



# ALE vertical coordinates

S. Danilov et al. 2017

- The layer-integrated momentum equation in the flux form

$$\begin{aligned} \partial_t(h\mathbf{u}) + \nabla \cdot (h\mathbf{u}\mathbf{u}) + w^t\mathbf{u}^t - w^b\mathbf{u}^b + f\mathbf{k} \times \mathbf{u}h \\ + h(\nabla p + g\rho\nabla Z) / \rho_0 \\ = D_u h\mathbf{u} + (\nu_v \partial_z \mathbf{u})^t - (\nu_v \partial_z \mathbf{u})^b, \end{aligned} \quad (6)$$

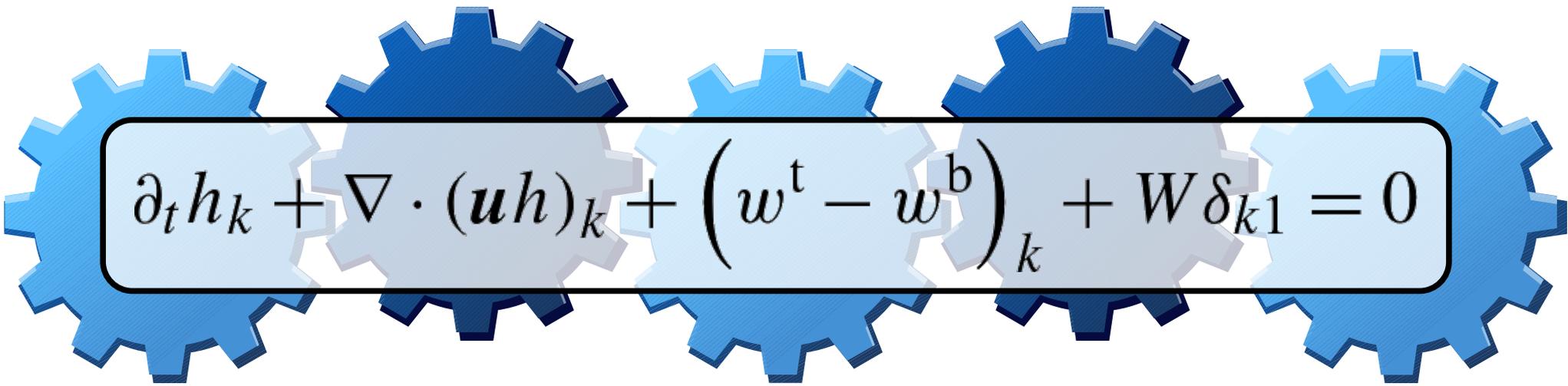


# ALE vertical coordinates

## How to implement different vertical ALE coordinates ?

- The different vertical ALE coordinates for the ...
  - linear free surface (all layers are fixed) and ...
  - full free surface (layer can move vertically)

mode are implemented via the continuity equation for the layer thickness and the resulting calculation of the vertical velocity


$$\partial_t h_k + \nabla \cdot (\mathbf{u}h)_k + (w^t - w^b)_k + W \delta_{k1} = 0$$



# ALE vertical coordinates



## ■ Asynchronous time stepping

- FESOM 1.4 uses asynchronous time stepping → horizontal velocities and scalars shifted by a half time step.
- adapt to FESOM2.0 → elevation (+velocities) defined at full-time levels, layer thicknesses (+tracers) defined at half-time levels

$$\begin{aligned}
 & h^{n+1/2} - h^{n-1/2} \\
 & = -\tau \left[ \nabla \cdot (\mathbf{u}^n h^*) + w^t - w^b + W^{n-1/2} \delta_{k1} \right] \quad (9)
 \end{aligned}$$

$$\begin{aligned}
 & h^{n+1/2} T^{n+1/2} - h^{n-1/2} T^{n-1/2} = -\tau \left[ \nabla \cdot (\mathbf{u}^n h^* T^n) \right. \\
 & \left. + w^t T^t - w^b T^b + W^{n-1/2} T_W \delta_{k1} \right] + D_T \quad (10)
 \end{aligned}$$



# ALE vertical coordinates



- The elevation at full time steps and the total thickness on half-steps, given by the vertical sum of  $h_k$ , may become decoupled due to numerical errors.
- suppress decoupling, by maintaining consistency between the physical layer thickness ( $h$ , used with tracers) and dynamical thickness (dependent on the elevation  $\eta$ ).

$$\bar{h} = \sum_k h_k - H, \quad (11)$$

- $H$  ... is the unperturbed ocean thickness.

$\bar{h}$  ... elevation from derived from thickness  $\rightarrow$  would be identical to the elevation  $\eta$  in the continuous world, but not in the discrete formulation here



# ALE vertical coordinates

- write for the elevation

$$\eta^{n+1} - \eta^n = -\tau \left( \alpha \left( \nabla \cdot \int_{\bar{h}^{n+1/2}} \mathbf{u}^{n+1} dz + W^{n+1/2} \right) + (1 - \alpha) \left( \nabla \cdot \int_{\bar{h}^{n-1/2}} \mathbf{u}^n dz + W^{n-1/2} \right) \right). \quad (12)$$

- Eq. for thicknesses can be vertically integrated (with condition that the surface value of  $\omega^t = 0$  vanishes

$$\bar{h}^{n+1/2} - \bar{h}^{n-1/2} = -\tau \nabla \cdot \int_{\bar{h}^{n-1/2}} \mathbf{u}^n dz - \tau W^{n-1/2}. \quad (13)$$



# ALE vertical coordinates



- Expressing the rhs in the formula for  $\eta$  through the difference in surface displacements  $h \rightarrow \eta$  and  $\bar{h}$  can be made consistent if

$$\eta^n = \alpha \bar{h}^{n+1/2} + (1 - \alpha) \bar{h}^{n-1/2}. \quad (14)$$

- To avoid diverging of  $\eta$  and  $\bar{h}$ :
  - Compute  $\eta^n$  from eq. 14
  - Estimate  $\eta^{n+1}$  from dynamical equation (stiffness matrix needs to be updated every time step) use only to compute  $u^{n+1}$
  - on new time step a “copy” of  $\eta_{n+1}$  will be created from the respective fields  $\bar{h}$  of eq. 14



# Solution strategy

## ALE vertical coordinates



- Compute  $\eta^n$  from  $\bar{h}$  by...

$$\eta^n = \alpha \bar{h}^{n+1/2} + (1 - \alpha) \bar{h}^{n-1/2} \quad \bar{h} = \sum_k h_k - H$$

- Actualize stiffness matrix to calculate elevation  $\eta$
- Compute RHS of elevation equation and solve for elevation  
→  $d\eta = \eta^{n+1} - \eta^n$

- Update horizontal velocity:

$$\mathbf{u}^{n+1} - \mathbf{u}^* = -g\tau\theta\nabla \left( \eta^{n+1} - \eta^n \right)$$

- Compute  $\bar{h}^{n+3/2}$  with actualized horizontal velocities from...

$$\bar{h}^{n+1/2} - \bar{h}^{n-1/2} = -\tau\nabla \cdot \int \mathbf{u}^n dz - \tau W^{n-1/2}$$



# Solution strategy

## ALE vertical coordinates



- Calculate layer thicknesses and  $\omega$  from continuity equation with different options for: linfs, zlevel and zstar ...

$$\partial_t h_k + \nabla \cdot (\mathbf{u}h)_k + \left( w^t - w^b \right)_k + W \delta_{k1} = 0$$

- Calculate tracer advection and diffusion.

